ECL 4340

POWER SYSTEMS

LECTURE 18
UNBALANCED FAULT ANALYSIS

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ANNOUNCEMENTS

- Be reading Chapters 8 and 9
- HW 9 is uploaded, due November 11, Friday.
- Exam II is on November 8, Tuesday.

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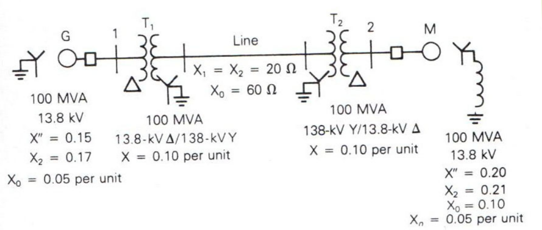
EXAMPLE 9.3



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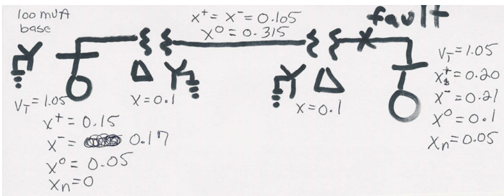
Unbalanced Fault Analysis

- The first step in the analysis of unbalanced faults is to assemble the three sequence networks. For example, for the earlier single generator and single motor example let's develop the sequence networks



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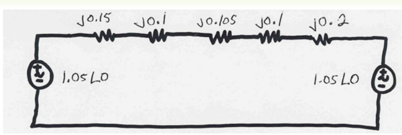
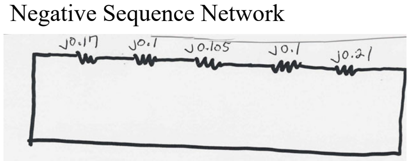
Sequence Diagrams for Example

Positive Sequence Network



Negative Sequence Network



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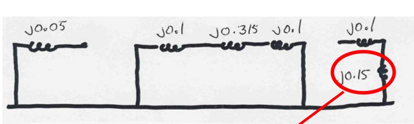
Sequence Diagrams for Example

Zero Sequence Network



Since zero sequence current is equal for all three phases, $I_{a0} = I_{b0} = I_{c0} = I_0$, current in the neutral impedance Z_n is $I_n = I_{a0} + I_{b0} + I_{c0} = 3I_0$, and voltage drop across Z_n is $Z_n(3I_0) = (3Z_n)I_0$

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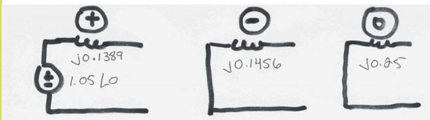
Create Thevenin Equivalents

- To do further analysis we first need to calculate the Thevenin equivalents as seen from the fault location. In this example the fault is at the terminal of the right machine so the Thevenin equivalents are:



$Z_{th}^{+} = j0.2$ in parallel with j0.455

$Z_{th}^{-} = j0.21$ in parallel with j0.475

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Single Line-to-Ground (SLG) Faults

- Unbalanced faults unbalance the network, but only at the fault location. This causes a coupling of the sequence networks. How the sequence networks are coupled depends upon the fault type. We'll derive these relationships for several common faults.
- With a SLG fault, only one phase has non-zero fault current -- we'll assume it is phase A.

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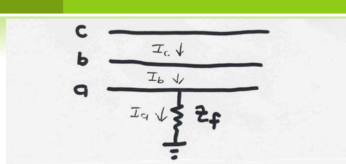
SLG Faults, cont'd



$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} ? \\ 0 \\ 0 \end{bmatrix}$$

Then since

$$\begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a^f \\ 0 \\ 0 \end{bmatrix} \rightarrow \frac{I_f^0 = I_f^+ = I_f^- = \frac{1}{3} I_a^f}{I_a^f = 3 I_f^0} \quad (1)$$

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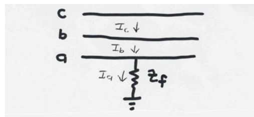
SLG Faults, cont'd

$$V_a^f = Z_f I_a^f = Z_f\left(3I_f^0\right) = 3Z_f I_f^0$$

$$\begin{bmatrix} V_a^f \\ V_b^f \\ V_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix}$$



This means $V_a^f = V_f^0 + V_f^+ + V_f^- = 3Z_f I_f^0$ (2)

The only way these two constraints (1) & (2) can be satisified is by coupling the sequence networks in series

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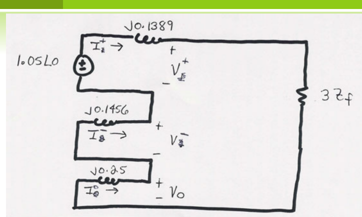
SLG Faults, cont'd



With the sequence networks in series we can solve for the fault currents (assume Z_f=0)

$$I_f^+ = \frac{1.05\angle 0^\circ}{j(0.1389 + 0.1456 + 0.25 + 3Z_f)} = -j1.964 = I_f^- = I_f^0$$

$$\mathbf{I} = \mathbf{A}\,\mathbf{I}_s \rightarrow I_a^f = -j5.8 \text{ (of course, } I_b^f = I_c^f = 0)$$

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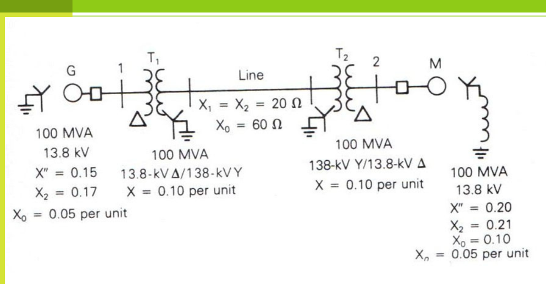
Example 9.3



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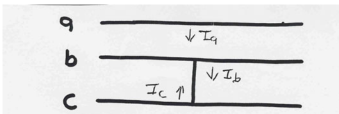
Line-to-Line (LL) Faults

- The second most common fault is line-to-line, which occurs when two of the conductors come in contact with each other. Without loss of generality we'll assume phases *b* and *c*.



Current Relationships: $I_a^f = 0, \quad I_b^f = -I_c^f$

Voltage Relationships: $V_{bg} = V_{cg}$

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LL Faults, cont'd

Using the current relationships we get

$$\begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b^f \\ -I_b^f \end{bmatrix} \rightarrow$$

$$I_f^0 = 0$$

$$I_f^+ = \frac{1}{3} I_b^f\left(\alpha - \alpha^2\right) \qquad I_f^- = \frac{1}{3} I_b^f\left(\alpha^2 - \alpha\right)$$

$$\text{Hence } I_f^+ = -I_f^- \qquad (1)$$

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LL Faults, con'td

Using the voltage relationships we get

$$\begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_{ag}^f \\ V_{bg}^f \\ V_{cg}^f \end{bmatrix}, \quad \text{where } V_{bg}^f = V_{cg}^f \quad \rightarrow$$

Hence

$$V_f^+ = \frac{1}{3}\left[V_{ag}^f + \left(\alpha + \alpha^2\right)V_{bg}^f\right]$$

$$V_f^- = \frac{1}{3}\left[V_{ag}^f + \left(\alpha^2 + \alpha\right)V_{bg}^f\right] \quad \rightarrow \quad V_f^+ = V_f^- \quad (2)$$

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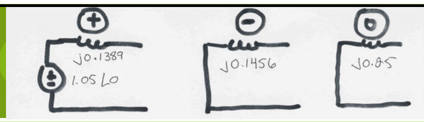
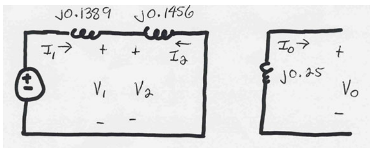


CONT'D

To satisfy $I_f^+ = -I_f^-$ & $V_f^+ = V_f^-$
the positive and negative sequence networks must be connected in parallel



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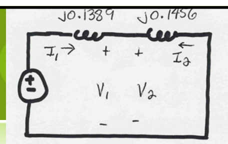

LL FAULTS, CONT'D

Solving the network for the currents we get

$$I_f^+ = \frac{1.05\angle 0°}{j0.1389 + j0.1456} = 3.691\angle -90°$$

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 3.691\angle -90° \\ 3.691\angle 90° \end{bmatrix} = \begin{bmatrix} 0 \\ -6.39 \\ 6.39 \end{bmatrix}$$

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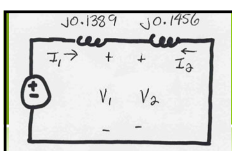

LL FAULTS, CONT'D

Solving the network for the voltages we get

$V_f^+ = 1.05\angle 0° - j0.1389 \times 3.691\angle -90° = 0.537\angle 0°$

$V_f^- = -j0.1456 \times 3.691\angle 90° = 0.537\angle 0°$

$$\begin{bmatrix} V_a^f \\ V_b^f \\ V_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.537 \\ 0.537 \end{bmatrix} = \begin{bmatrix} 1.074 \\ -0.537 \\ -0.537 \end{bmatrix}$$

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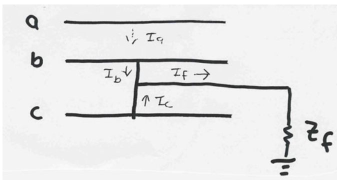
Double Line-to-Ground Faults

- With a double line-to-ground (DLG) fault, two-line conductors come in contact both with each other and ground. We'll assume these are phases *b* and *c*.



$I_a^f = 0 \qquad V_{bg}^f = V_{cg}^f = Z_f(I_b^f + I_c^f)$

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DLG Faults, cont'd

From the current relationships we get

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix}$$

Since $I_a^f = 0 \quad \rightarrow \quad I_f^0 + I_f^+ + I_f^- = 0 \quad (1)$

Note, because of the path to ground, the zero sequence current is no longer zero.

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DLG Faults, cont'd

From the voltage relationships we get

$$\begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_{ag}^f \\ V_{bg}^f \\ V_{bg}^f \end{bmatrix} \rightarrow V_f^+ = V_f^- \qquad (2)$$

Now, $\begin{bmatrix} V_{ag}^f \\ V_{bg}^f \\ V_{cg}^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix}$

Then $V_{bg}^f = V_f^0 + (\alpha^2 + \alpha) V_f^+$

But since $1 + \alpha + \alpha^2 = 0 \quad \rightarrow \quad \alpha^2 + \alpha = -1$

$V_{bg}^f = V_f^0 - V_f^+$

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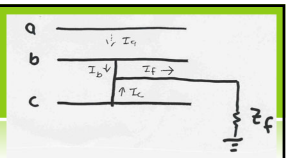
Now, $V_{bg}^f = V_f^0 - V_f^+$

$= Z_f(I_b^f + I_c^f)$



Also, since

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix}$$

$$I_b^f = I_f^0 + \alpha^2 I_f^+ + \alpha I_f^-$$

$$I_c^f = I_f^0 + \alpha I_f^+ + \alpha^2 I_f^-$$

Adding these together (with $\alpha + \alpha^2 = -1$)

$$V_{bg}^f = Z_f(2I_f^0 - I_f^+ - I_f^-) \quad \text{with } I_f^0 = -I_f^+ - I_f^-$$

$$V_f^0 - V_f^+ = 3Z_f I_f^0 \qquad (3)$$

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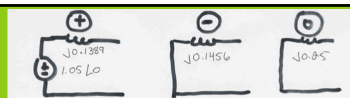
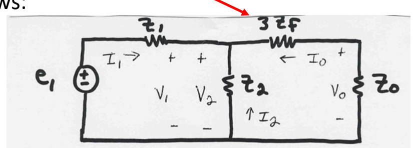
$V_f^0 - V_f^+ = 3Z_f I_f^0$



- The three sequence networks are joined as follows:



Assuming Z_f=0, then

$$I_f^+ = \frac{V^+}{Z^+ + Z^- // (Z^0 + 3Z_f)} = \frac{1.05\angle 0°}{j0.1389 + j0.092}$$

$$= 4.547\angle -90°$$

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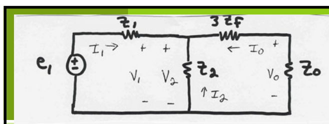

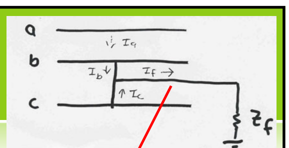

$V_f^+ = 1.05 - 4.547\angle -90° \times j0.1389 = 0.4184$

$I_f^- = -0.4184 / j0.1456 = j2.874$

$I_f^0 = -I_f^+ - I_f^- = j4.547 - j2.874 = j1.673$

Converting to phase: $I_b^f = -1.04 + j6.82$

$I_c^f = 1.04 + j6.82$

Fault current: $I^f = I_b^f + I_c^f = j13.64$

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Unbalanced Fault Summary

- SLG: Sequence networks are connected in series, parallel to three times the fault impedance
- LL: Positive and negative sequence networks are connected in parallel; zero sequence network is not included since there is no path to ground
- DLG: Positive, negative and zero sequence networks are connected in parallel, with the zero sequence network including three times the fault impedance

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Generalized System Solution

- Assume we know the pre-fault voltages
- The general procedure is then

1. Calculate Z_{bus} for each sequence
2. For a fault at bus *i*, the Z_{ii} values are the Thevenin equivalent impedances; the pre-fault voltage is the positive sequence Thevenin voltage
3. Connect and solve the Thevenin equivalent sequence networks to determine the fault current

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Generalized System Solution, cont'd

4. Sequence voltages throughout the system are given by

$$\mathbf{V} = \mathbf{V}^{prefault} + Z_{bus} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -I_f \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

This is solved for each sequence network!

5. Phase values are determined from the sequence values

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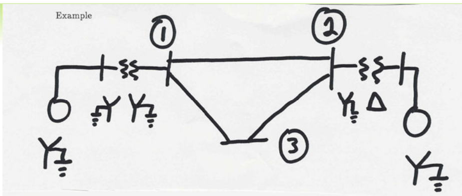
Unbalanced System Example



For the generators assume $Z^+ = Z^- = j0.2$; $Z^0 = j0.05$
For the transformers assume $Z^+ = Z^- = Z^0 = j0.05$
For the lines assume $Z^+ = Z^- = j0.1$; $Z^0 = j0.3$
Assume unloaded pre-fault, with voltages =1.0 p.u.

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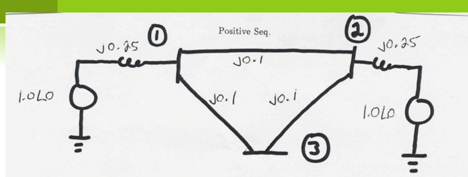
Positive/Negative Sequence Network



$$\mathbf{Y}_{bus}^{+} = j\begin{bmatrix} -24 & 10 & 10 \\ 10 & -24 & 10 \\ 10 & 10 & -20 \end{bmatrix} \quad \mathbf{Z}_{bus}^{+} = j\begin{bmatrix} 0.1397 & 0.1103 & 0.125 \\ 0.1103 & 0.1397 & 0.125 \\ 0.1250 & 0.1250 & 0.175 \end{bmatrix}$$

Negative sequence is identical to positive sequence

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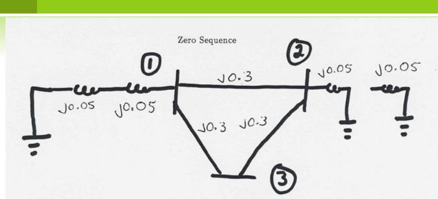
Zero Sequence Network



$$\mathbf{Y}_{bus}^{0} = j\begin{bmatrix} -16.66 & 3.33 & 3.33 \\ 3.33 & -26.66 & 3.33 \\ 3.33 & 3.33 & -6.66 \end{bmatrix} \quad \mathbf{Z}_{bus}^{0} = j\begin{bmatrix} 0.0732 & 0.0148 & 0.0440 \\ 0.0148 & 0.0435 & 0.0292 \\ 0.0440 & 0.0292 & 0.1866 \end{bmatrix}$$

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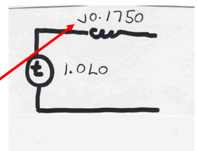
For a SLG Fault at Bus 3

The sequence networks are created using the pre-fault voltage for the positive sequence Thevenin voltage, and the Z_{bus} diagonals for the Thevenin impedances

$$\mathbf{Z}_{bus}^{+} = j\begin{bmatrix} 0.1397 & 0.1103 & 0.125 \\ 0.1103 & 0.1397 & 0.125 \\ 0.1250 & 0.1250 & \boxed{0.175} \end{bmatrix}$$



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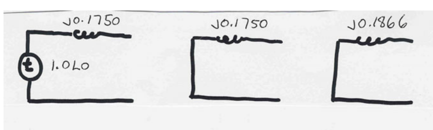
For a SLG Fault at Bus 3

Repeat for the negative and zero sequence networks,

Positive Seq. Negative Seq. Zero Seq.



The fault type then determines how the networks are interconnected

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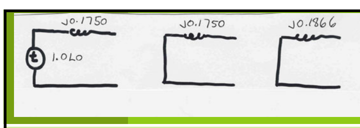

Fault, cont'd

$$I_f^{+} = \frac{1.0\angle 0^\circ}{j(0.1750+0.1750+0.1866)} = -j1.863$$

$$I_f^{+} = I_f^{-} = I_f^{0} = -j1.863$$

$$\mathbf{V}^{+} = \begin{bmatrix} 1.0\angle 0^\circ \\ 1.0\angle 0^\circ \\ 1.0\angle 0^\circ \end{bmatrix} + \mathbf{Z}_{bus}^{+}\begin{bmatrix} 0 \\ 0 \\ \boxed{j1.863} \end{bmatrix} = \begin{bmatrix} 0.7671 \\ 0.7671 \\ 0.6740 \end{bmatrix}$$

$$\mathbf{V}^{-} = \mathbf{Z}_{bus}^{-}\begin{bmatrix} 0 \\ 0 \\ j1.863 \end{bmatrix} = \begin{bmatrix} -0.2329 \\ -0.2329 \\ -0.3260 \end{bmatrix}$$

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Bus 3 SLG Fault, cont'd

$$\mathbf{V}^0 = \mathbf{Z}^0_{bus}\begin{bmatrix} 0 \\ 0 \\ j1.863 \end{bmatrix} = \begin{bmatrix} -0.0820 \\ -0.0544 \\ -0.3479 \end{bmatrix}$$

We can then calculate the phase voltages at any bus

$$\mathbf{V}_3 = \mathbf{A}\times\begin{bmatrix} -0.3479 \\ 0.6740 \\ -0.3260 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.522 - j0.866 \\ -0.522 + j0.866 \end{bmatrix}$$

$$\mathbf{V}_1 = \mathbf{A}\times\begin{bmatrix} -0.0820 \\ 0.7671 \\ -0.2329 \end{bmatrix} = \begin{bmatrix} 0.4522 \\ -0.3491 - j0.866 \\ -0.3491 + j0.866 \end{bmatrix}$$

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Faults on Lines

- The previous analysis has assumed that the fault is at a bus. Most faults occur on transmission lines, not at the buses
- For analysis, these faults are treated by including a dummy bus at the fault location. Now the impedance of the transmission line is then split depends upon the fault location

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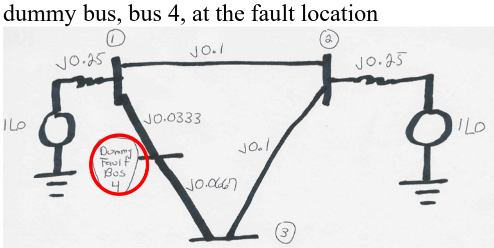
Line Fault Example

Assume a SLG fault occurs on the previous system on the line from bus 1 to bus 3, one third of the way from bus 1 to bus 3. To solve the system, we add a dummy bus, bus 4, at the fault location



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$$\mathbf{Y}_{bus}^{+} = j\begin{bmatrix} -24 & 10 & 10 \\ 10 & -24 & 10 \\ 10 & 10 & -20 \end{bmatrix} \quad \mathbf{Z}_{bus}^{+} = j\begin{bmatrix} 0.1397 & 0.1103 & 0.125 \\ 0.1103 & 0.1397 & 0.125 \\ 0.1250 & 0.1250 & 0.175 \end{bmatrix}$$

The Y_{bus} now has 4 buses

$$\mathbf{Y}_{bus}^{+} = j\begin{bmatrix} -44 & 10 & 0 & 30 \\ 10 & -24 & 10 & 0 \\ 0 & 10 & -25 & 15 \\ 30 & 0 & 15 & -45 \end{bmatrix}$$

Adding the dummy bus only changes the new row/column entries associated with the dummy bus

$$\mathbf{Z}_{bus}^{+} = j\begin{bmatrix} 0.1397 & 0.1103 & 0.1250 & 0.1348 \\ 0.1103 & 0.1397 & 0.1250 & 0.1152 \\ 0.1250 & 0.1250 & 0.1750 & 0.1417 \\ 0.1348 & 0.1152 & 0.1417 & 0.1593 \end{bmatrix}$$

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